On the Ambiguity of Job Search

Ying Tung Chan and Chi Man Yip*

Abstract

This paper generalizes the Diamond-Mortensen-Pissarides (DMP) model by incorporating ambiguity preferences. Our analytically tractable model preserves most of the comparative statics results in the DMP model. Ambiguity-averse workers believe that lower match-specific productivity levels are more likely realized, lowering their reservation wage. This belief makes them more likely to accept a contract from firms, reducing unemployment. We quantify unemployment attributable to these ambiguity preferences, namely ambiguous unemployment. We show that unemployment could have increased if both workers and firms became ambiguity-neutral. This ambiguous unemployment is countercyclical and can reach 19 percent of actual unemployment in the United States. Our model shows that ambiguity preferences can explain why the strong correlation between unemployment and volatility shocks to a productivity distribution is weakened in slumps.

Keywords: Ambiguity Preferences; Ambiguous Unemployment; Volatility Shocks.

JEL Classification Numbers: D81, E24, E32, E7, J64.

*Chan: The Research Institute of Economics and Management, Southwestern University of Finance and Economics, Chengdu, China (email: chanyt@swufe.edu.cn); Yip: Department of Economics, University of Calgary, 2500 University Dr. N.W., Calgary, AB, T2N 1N4 (email: cman.yip@gmail.com). We would like to thank Omid Atabati, Yu Chen, Mojtaba Eslami, David Green, Fabian Lange, Charles Ka Yui Leung, Jian Li, Ngo Van Long, Ioana Marinescu, Fruzsina Mayer, Dimitri Migrow, Theodore Papageorgiou, Meng Sun, Qian Sun, Atsuko Tanaka, Scott Taylor, Trevor Tombe, and seminar participants at the Fourth SOLE/EALE World Conference, the CLSRN Annual Conference, and the University of Calgary for their invaluable comments.

1 Introduction

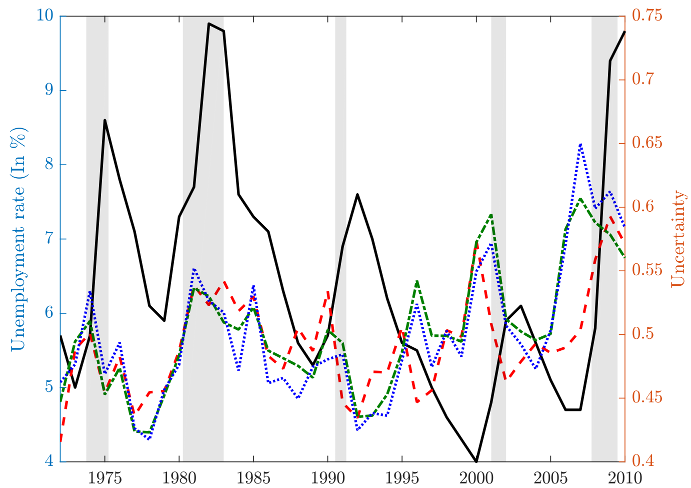
Prior literature assumes a common knowledge about a productivity distribution (Rogerson et al., 2005), yet this assumption is far from realistic. Workers and firms inside a model (economists construct) make decisions based on a specified productivity distribution, creating at least two sources of uncertainties. First, workers and firms are uncertain about which alternative model (e.g., the distribution function) should be used to make decisions. Since the true model is unknown, another uncertainty from model misspecification arises: the underlying model they use to make decision is potentially misspecified.

This paper constructs a model that allows us to investigate and quantify the impacts of these ambiguity preferences on labor market outcomes. The conditional mean of a productivity distribution above a reservation productivity threshold is important in determining expected profits, which affect decisions pertaining to the creation of vacancies and thus unemployment. Nevertheless, the underlying distribution function of productivity and thus the corresponding conditional mean are unknown, and we know little about how an aversion to this ambiguity and a fear about model misspecification affect the behaviours of workers and firms during a job search process. This paper purposes to (i) construct a search-theoretical model featuring ambiguity preferences, (ii) uncover the major mechanisms through which ambiguity preferences affect labor market outcomes, (iii) quantify unemployment attributable to these ambiguity preferences, and (iv) resolve the puzzle concerning the relationships between unemployment and volatility shocks to a productivity distribution.

This paper contributes to the literature on search-theoretical models. It develops an analytically tractable version of the Diamond-Mortensen-Pissarides (DMP) model featuring the ambiguity preferences of workers and firms (Hansen and Sargent, 2008).[1] While this literature often assumes ambiguity-neutral, we show that the DMP model is a special case of our model in which workers and firms are ambiguity-neutral. Our generalization allows us to uncover mechanisms through which ambiguity preferences affect important macroeconomic variables such as unemployment. We analytically show that our model preserves most (if not all) of the intuitive comparative statics results in the DMP model. As known, the DMP model, serving as a canonical search-theoretical model, can be easily extended to incorporate other labor market features such as on-the-job searches (Dolado et al., 2009; Postel-Vinay and Turon, 2014), firing costs (Postel-Vinay and Turon, 2014; Vindigni et al., 2014), job referral (Calvó-Armengol and Zenou, 2005; Galenianos, 2014), heterogeneity in sectors (Acemoglu, 2001; Albrecht et al., 2018), discrimination (Sasaki, 1999; Rosén, 2003), human capital accumulation (Cairo and Cajner, 2018), etc. Hence, our extension of

[1] We will compare our model with the one under Knightian uncertainty as in Nishimura and Ozaki (2004) in Online Appendix A.

Figure 1: Unemployment Rate and Uncertainty in the United States



Notes: The solid line shows the annual unemployment rate in the United States. The dashed line, dashed-dot line, and dotted-line show the standard deviation of total factor productivity for manufacturing establishments aged at least 2, 25, and 38 years, respectively. The shaped years represent the recession. Data on the standard deviations is obtained from Bloom et al. (2018).

the DMP model complements these broad literatures.

Our analytical result shows that the impacts of ambiguity preferences on unemployment could be different. Workers with stronger ambiguity aversion tend to believe that lower productivity levels are more likely to be realized. This belief reduces an unemployed worker's outside option value and thus a reservation wage; hence, they are more likely to accept a contract from firms. Meanwhile, the lower reservation wage requires firms to compensate employees less, thereby reducing wages. The reduction in this compensation increases expected profits, encouraging the creation of vacancies. Both the decrease in the reservation wage and the increase in the supplies of vacancies reduce unemployment. Whereas conventional wisdom suggests that complete information on the labor market helps job seekers get rid of unemployment, our result reveals that the ambiguity aversion of worker in fact reduces unemployment.

In contrast, firms with stronger ambiguity aversion formulate a belief of lower expected profits, discouraging their creation of vacancies. This belief simultaneously reduces the reservation productivity threshold to hire a worker. While the former effect increases unemployment, the latter effect shortens an average unemployment spell. These two compelling forces make it uncertain how a firm's ambiguity aversion affects unemployment.

Another contribution of this paper is to quantify unemployment attributable to ambiguity preferences, namely ambiguous unemployment. Our quantitative analysis indicates that unemployment would have increased if both workers and firms became ambiguity-neutral

Table 1: Correlation Coefficients between the Unemployment Rate and TFP Volatility

	Overall	**Recession**	**Non-Recession**
S.D. of TFP (Age: 2+ Years)	03998	0.3378	0.4094
S.D. of TFP (Age: 25+ Years)	0.1714	0.0345	0.2077
S.D. of TFP (Age: 38+ Years)	0.3004	0.1317	0.3671

Notes: This table shows the correlation coefficients between the annual unemployment rate and the one-year lagged standard deviation of total factor productivity for the corresponding manufacturing firms in the United States during 1972-2010. Data on the standard deviations is obtained from Bloom et al. (2018), and the unemployment rate is obtained from the Federal Bank of St. Louis.

in reality. This ambiguous unemployment is countercyclical, and its size can be considerable: it can reach 19 percent of actual unemployment in the United States. Furthermore, the implications of this analysis lead to an interesting policy dilemma in propagating labor market information. We find that the removal of the firm's ambiguity reduces unemployment, but the impact is negligible. In contrast, if the worker's ambiguity is removed, the effect is substantial but unemployment increases.

This paper also complements a series of influential papers that study the relationship between unemployment and productivity uncertainties (Bloom et al., 2007; Bloom, 2009; Schaal, 2017; Bloom et al., 2018). Figure 1 plots the unemployment rate and the standard deviation of the total factor productivity (TFP) in the United States during 1972-2010. Apparently, these two variables are large in slumps. Table 1 reports the correlation coefficients between the unemployment rate and three different measures of one-year lagged standard deviation of the TFP.

Two points deserve mentioning. First, all the correlation coefficients are positive; volatility shocks are positively correlated with unemployment rates. This result coheres with Schaal (2017), which shows that U.S. unemployment fluctuations could be explained in large part by the volatility shocks to the TFP. Second, these coefficients are weaker during periods of recession regardless of the volatility measure. For example, using the firms aged 25 and 38 years more, the correlation coefficients during recession years are about one-fifth and one-third of their counterparts during non-recession years, respectively. No prior literature explains this phenomenon.

This paper provides a rationale behind the phenomenon. Our quantitative analysis reveals that a larger standard deviation of the productivity distribution could catalyze ambiguous unemployment. Since both ambiguous unemployment and the standard deviation of the productivity distribution are procyclical, ambiguity preferences reduce unemployment more in slumps than in booms, weakening the strong correlation between unemployment and volatility shocks in recession years. This paper contributes to this literature by linking ambiguity preferences to the relationship between unemployment and the volatility of a productivity distribution. This linkage, though important in understanding unemployment fluctuations, has not been addressed in previous formal models.

This paper is organized as follows. Section 2 presents a basic model setting. Section 3 characterizes a steady-state equilibrium and explores the impacts of ambiguity preferences on the labor market. Comparative statics are analytically shown in Section 4. Section 5 quantifies and explores the properties of ambiguous unemployment and shows that ambiguity unemployment could explain why the strong correlation between unemployment and volatility shocks is weakened in recession years. Section 6 concludes this paper.

2 The Basic Model

This section constructs a search-theoretical model that allows both workers and vacancies to be ambiguity-averse.[2] In contrast to the literature, we do not assume that workers and vacancies know the data-generating process of an economy. They utilize an approximating model that best describes the data-generating process. Meanwhile, they fear possible misspecification of this approximating model. Consequently, they do not make decisions simply according to the rational expectation paradigm. Instead, they incorporate a penalty function into their value functions to diverge from the approximating model; they distort the approximating model so that the fear of model misspecification can be considered. In addition to the decisions on job searchs and vacancy creations as in the conventional search and matching model, the model of choice is endogenized so that the decisions during job/worker search processes are made to maximize the value function under the worst-case scenario. Following the convention in the literature on ambiguity preferences, we call the optimal model, which is chosen by an ambiguity-averse agent, a distorted model.

Consider a discrete time economy with a fixed labor force, without loss of generality normalized to unity. Each worker has an infinite horizon and is either employed or unemployed.[3] A vacancy is either filled or unfilled. The measure of the vacancy is endogenized. Both workers and vacancies are ambiguity-averse, know the degree of ambiguity aversion of each other, and share an identical discounted factor $\beta \equiv 1/(1+r)$, where r is a real

[2] To shed light on the impacts of ambiguity preferences on other labor market outcomes, a search and matching model is constructed at its simplest. Readers who are interested in the model with other elements such as learning are referred to Moscarini (2005), Gonzalez and Shi (2010), and Papageorgiou (2014) for a search and matching model with learning and Epstein and Schneider (2008) for the ambiguity preferences with learning.

[3] The model assumes that no decision on labor supply, either the number of working hours or labor force participation, is made. This simplification is standard (Mortensen and Pissarides, 1994; Shimer, 2005; Hall, 2005; Hagedorn and Manovskii, 2008; Hall and Milgrom, 2008; Fujita and Ramey, 2012; Michaillat, 2012), and is in line with empirical regularities: cyclical variations in total working hours (unemployment) basically arise from changes in the amount of employment but not changes in working hours per worker (labor force participation) (Shimer, 2010).

interest rate.

An unemployed worker receives unemployment benefit b and meets an unfilled vacancy via a matching technology $M(u, v)$, where u and v are the numbers of unemployed workers and unfilled vacancies, respectively. The matching technology $M(u, v)$ gives the number of pairwise meetings per period. Assume that $M(u, v)$ is homogeneous of degree one and increasing in each argument so that $M(u, v)/v = M(u/v, 1)$. We denote the rate at which an unfilled vacancy meets a job seeker $q(\theta) \equiv M(u, v)/v$, where $\theta \equiv v/u$ is the market tightness. It follows that $q(\theta)$ is a decreasing function and that the rate at which a job seeker meets an unfilled vacancy $M(u, v)/u = \theta q(\theta)$ is an increasing function of θ, as in Pissarides (2000). We also make standard Inada-type assumptions on $M(u, v)$ so that $\lim_{\theta\to 0} q(\theta) = \infty$, $\lim_{\theta\to\infty} q(\theta) = 0$, $\lim_{\theta\to 0} \theta q(\theta) = 0$, and $\lim_{\theta\to\infty} \theta q(\theta) = \infty$.

When an unemployed worker and an unfilled vacancy meet, a match-specific productivity level δ is realized. We assume δ follows a cumulative distribution function $F(\delta)$ over $\mathbb{R}_+$, and the corresponding probability density function is denoted by $f(\delta)$. Given δ, they bargain on the wage. The unemployed worker will reject the job offer if the expected lifetime utility that would result from remaining in the status of an unemployed job seeker exceeds that of employment with productivity δ.

In contrast to the literature, we do not assume that job seekers have full knowledge of the productivity distribution $F(\delta)$. Instead, these job seekers are not confident that $F(\delta)$ is the "true" distribution. As shown in Hansen and Sargent (2008), an ambiguity-averse agent will replace the density function $f(\delta)$ with an alternative density $\hat{f}(\delta)$ to account for his fear of losses arising from model misspecification. Following Hansen and Sargent (2008), this class of problem can be formulated as choosing the likelihood ratio $m(\delta) \equiv \hat{f}(\delta)/f(\delta)$ for all δ to optimize a value function that is a function of an ambiguity preference $\alpha \leq 0$ and a measure of the distance between the two distributions. We follow Hansen and Sargent (2008) to use $\mathbb{E}_\delta m(\delta) \ln m(\delta)$, known as relative entropy between $f(\delta)$ and $\hat{f}(\delta)$, to measure the Kullback Leibler distance between two distributions. In this model, this relative entropy measures the discrepancy between the approximating and the distorted model. Denote $J^E(\delta)$ and J^U as the value functions of employment and unemployment, respectively. The value function of an unemployed worker J^U must satisfy the following function:

$$
\begin{aligned}
J^U &= \min_{m(x)} b + \beta \mathbb{E}_x \left[\theta q(\theta) \left(m(x) \max\{J^E(x), J^U\} - \frac{1}{\alpha} m(x) \ln m(x) \right) + (1 - \theta q(\theta)) J^U \right] \\
\text{subject to} &\quad \int m(x) dF(x) = 1, \qquad (1)
\end{aligned}
$$

where $m(\delta) \equiv \hat{f}(\delta)/f(\delta)$ is a likelihood ratio. An unemployed worker chooses a likelihood ratio to minimize the value function, and the decisions during a job search process are made to maximize this minimized value function. We can also interpret the choice of $m(\delta)$ as follows. Given the approximating model $f(\delta)$, an unemployed worker chooses the distorted model $\hat{f}(\delta)$ to minimize the value function. The decisions are then made to maximize the value function under the worst-case scenario in which the probability density function is $\hat{f}(\delta)$.

$\alpha \leq 0$ is a penalty parameter for relative entropy, which captures the degree of a worker's ambiguity aversion. When $\alpha = 0$, workers are said to be ambiguity-neutral. The more workers fear model misspecification, the lower α becomes.

The optimal likelihood ratio is given by[4]

$$\frac{\hat{f}(\delta)}{f(\delta)} = \frac{e^{\alpha \max\{J^E(\delta), J^U\}}}{\int_0^\infty e^{\alpha \max\{J^E(x), J^U\}} dF(x)}. \tag{2}$$

We will show that the partial derivative $\partial J^E(\delta)/\partial\delta$ is positive. Hence, the optimal likelihood ratio decreases with δ. Intuitively, an ambiguity-averse job seeker will choose the "distorted distribution" that assigns a lower probability to a higher match-specific productivity level. In the limiting case where α approaches zero from below, equation (2) implies that $\hat{f}(\delta) = f(\delta)$, and thus

$$J^U = b + \beta \mathbb{E}_x \bigg(\theta q(\theta) \max\{J^E(x), J^U\} + (1 - \theta q(\theta)) J^U \bigg),$$

which is the case in the absence of the fear of model misspecification. When a worker does not worry about model misspecification, his belief of the probability density function is identical to the approximating one. Hence, the value function of unemployment in the conventional search and matching model is a special case of ours when α equals zero.

An employed worker with a match-specific productivity level δ receives a bargained wage $w(\delta)$ and, at the end of each period, faces a separation shock at a rate of λ. When the shock arrives, the worker becomes unemployed. The discounted present values of employment $J^E(\delta)$ can be written as follows:

$$J^E(\delta) = w(\delta) + \beta \bigg(\lambda J^U + (1 - \lambda) J^E(\delta) \bigg). \tag{3}$$

A filled vacancy generates a production value δ, pays a worker $w(\delta)$, and faces a separation shock at a rate of λ. When the shock arrives, a filled vacancy becomes unfilled. Denote $J^F(\delta)$ and J^V as asset values of a filled and an unfilled vacancy, respectively. The asset

[4] Proof is given in Appendix 7.1.

value of a filled vacancy $J^F(\delta)$ can be written as follows:

$$J^F(\delta) = \delta - w(\delta) + \beta\Big(\lambda J^V + (1-\lambda)J^F(\delta)\Big). \tag{4}$$

An unfilled vacancy pays maintenance cost $c > 0$ and faces a probability $q(\theta)$ of being filled. When an unfilled vacancy meets an unemployed worker, it will agree to form a match with the worker if the match-specific productivity level is high enough such that $J^F(\delta) \geq J^V$. An unfilled vacancy, being ambiguity-averse, maximizes the minimum expected outcome. To do so, a vacancy chooses $m_v(\delta) \equiv \hat{f}_v(\delta)/f(\delta)$, a likelihood ratio, to maximize the minimum expected outcome, where $\hat{f}_v(\delta)$ is a probability density function of the distorted model chosen by an unfilled vacancy. Hence, the asset value of an unfilled vacancy J^V can be written as

$$\begin{aligned} J^V &= \min_{m_v(x)} -c + \beta\mathbb{E}_x\left[q(\theta)\left(m_v(x)\max\{J^F(x), J^V\} - \frac{1}{\alpha_v}m_v(x)\ln m_v(x)\right) + (1-q(\theta))J^V\right] \\ \text{subject to} &\quad \int m_v(x)dF(x) = 1, \end{aligned} \tag{5}$$

where $\alpha_v \leq 0$ is the degree of ambiguity aversion of a vacancy. This problem can be solved in a similar way as the minimization problem facing unemployed workers (2). The corresponding likelihood ratio is given by

$$\frac{\hat{f}_v(\delta)}{f(\delta)} = \frac{e^{\alpha_v \max\{J^F(\delta), J^V\}}}{\int_0^\infty e^{\alpha_v \max\{J^F(x), J^V\}}dF(x)}. \tag{6}$$

The ambiguity preferences of workers and vacancies share the same notion in the optimal likelihood ratio: more ambiguity-averse agents tend to believe that a lower productivity level is more likely to be realized. When an unemployed worker and an unfilled worker meet, a match-specific productivity level δ is realized, and they bargain on the wage to maximize the generalized Nash product as follows:

$$w(\delta) \equiv \arg\max(J^E(\delta) - J^U)^\eta(J^F(\delta) - J^V)^{1-\eta},$$

where $\eta \in (0,1)$ is the bargaining power of workers.[5] Understanding the ambiguity preferences is important in wage determination because the degree of ambiguity aversion affects the bargained wage through the outside option values of workers and vacancies (i.e., J^U and J^V). For simplicity, we assume that the ambiguity preferences of workers and vacancies

[5] Maximizing this generalized Nash product is only one of the wage determination methods. So long as the wage determination method returns an employed worker a fraction of total matching rent, the ambiguity aversion of a worker can affect labor market outcomes. Therefore, the implications of this model is not driven by the proposed wage determination mechanism.

are common information.[6] Simple algebra gives the following sharing rule:

$$J^E(\delta) - J^U = \eta\Big(J^E(\delta) - J^U + J^F(\delta) - J^V\Big). \tag{7}$$

Intuitively, a match surplus of a worker is a fraction η of the total match surplus. The assumption of free entry and exit is made; hence, rent is exhausted, and thus

$$J^V = 0. \tag{8}$$

Denote δ_u^R as the reservation productivity level, below which an unemployed worker will not accept any job offer. Since an unemployed worker will accept the job offer only if $J^E(\delta) \geq J^U$, $J^E(\delta_u^R) = J^U$. A reservation productivity threshold for an unfilled vacancy δ_v^R is defined in a similar way; therefore, $J^F(\delta_v^R) = J^V$. Using the sharing rule (7), one can easily show that the reservation productivity levels for a worker and a vacancy are identical. Hereafter, denote the reservation productivity level as δ^R. Both $J^E(\delta)$ and $J^F(\delta)$ strictly increases with δ; therefore, the δ^R is unique. Hence, unemployed workers and unfilled vacancies, when they meet, accept the offer for all $\delta \geq \delta^R$. Using equations (3), (4), (7), and (8), the wage equation is derived as follows:

$$w(\delta) = \eta\delta + (1-\eta)\beta r J^U. \tag{9}$$

Hence, a wage is a fraction of production value plus a fraction of the worker's outside option value. Using equations (3) and (9), a reservation wage is given by

$$w(\delta^R) = \delta^R = \beta r J^U. \tag{10}$$

Hence, the reservation wage $w(\delta^R)$, the reservation productivity level δ^R, and the worker's outside option value are identical. If the realized productivity level δ exceeds δ^R, the bargained wage is given by

$$w(\delta) = (1-\eta)\delta^R + \eta\delta. \tag{11}$$

A worker is compensated with a fraction of the reservation productivity level δ^R plus a fraction of the production value.

A steady-state unemployment rate is determined by equating flows out and into unem-

[6]In reality, interviews enable vacancies to understand workers' match-specific productivity levels and attitudes toward ambiguity. For example, an expected salary on a resume could reveal an unemployed worker's belief of his/her expected productivity and/or ambiguity preference. Another example is an aptitude test, which could reveal the ambiguity preferences of workers. Readers who are interested in the model in which agents have asymmetric information on the degree of ambiguity aversion are referred to Ahn (2007).

ployment and is given by

$$u = \frac{\lambda}{\lambda + [1 - F(\delta^R)]\theta q(\theta)}. \tag{12}$$

A steady-state unemployment rate strictly increases with the reservation productivity level δ^R because a higher δ^R reduces a job offer acceptance rate and thus lengthens an unemployment spell.

3 Characterization of a Steady-State Equilibrium

This section characterizes a steady-state equilibrium and explores the impacts of ambiguity preferences on labor market outcomes. First, this section shows that our generalization of the DMP model is highly analytically tractable. Second, it uncovers the mechanisms through which ambiguity preferences affect the labor market in this stylized search and matching model. The empirical significance of these mechanisms will be presented in Section 5.

Definition 1. *A steady-state equilibrium is defined as* $\{\hat{f}(\delta), \hat{f}_v(\delta), \delta^R, w(\delta), u, \theta, J^E(\delta), J^U, J^F(\delta), J^V\}$ *such that equations (1), (3) to (5), (7), (8), (10), and (12) are satisfied for all* $\delta \geq \delta^R$*, and equations (2) and (6) are satisfied for all* $\delta \in \mathbb{R}_+$.

Using equations (1) and (2), the value function of an unemployed worker is written by

$$\beta r J^U = b + \frac{\beta\theta q(\theta)}{\alpha} \ln\left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\alpha(J^E(x) - J^U)} dF(x)\right). \tag{13}$$

Using equations (5) and (6), the value function of an unfilled vacancy is given by

$$\beta r J^V = -c + \frac{\beta q(\theta)}{\alpha_v} \ln\left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\alpha_v(J^F(x) - J^V)} dF(x)\right). \tag{14}$$

Notably, J^U strictly increases with θ, and J^V strictly decreases with θ, as commonly seen in the literature on the search and matching model. A rise in market tightness increases a transition rate from unemployment to employment but decreases the rate from being unfilled to being filled, thereby improving but deteriorating the value functions of an unemployed worker and an unfilled vacancy, respectively. When $\alpha = 0$ and $\alpha_v = 0$, the preceding equations reduce to

$$\begin{aligned} \beta r J^U &= b + \beta\theta q(\theta) \int_{\delta^R}^{\infty} (J^E(x) - J^U) dF(x) \text{ and} \\ \beta r J^V &= -c + \beta q(\theta) \int_{\delta^R}^{\infty} (J^F(x) - J^V) dF(x). \end{aligned}$$

They are the value functions of unemployed workers and unfilled vacancies in the model in which workers and vacancies are ambiguity-neutral.[7] Thus, the fundamental setup in the literature on the search equilibrium model is a special case of the present model, when both α and α_v equal zero. Interestingly, the four value functions (i.e., $J^E(\delta)$, $J^F(\delta)$, J^U, and J^V) preserve linearity only if both workers and vacancies are ambiguity-neutral (i.e., α and α_v are zero), making the search equilibrium model analytically tractable. We show that these value functions are all nonlinear with one another under ambiguity aversion. This section contributes to the literature by showing that even if these value functions are nonlinear under ambiguity aversion, the model preserves its analytical tractability.

Substituting equations (2), (3), (9), and (10) into equation (13), simple algebra gives

$$\delta^R = b + \frac{\beta\theta q(\theta)}{\alpha}\ln\left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}dF(x)\right). \tag{15}$$

Similarly, substituting equations (4), (6), (9), and (10) into equation (14) yields

$$c = \frac{\beta q(\theta)}{\alpha_v}\ln\left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha_v(1-\eta)(1+r)}{r+\lambda}(x-\delta^R)}dF(x)\right). \tag{16}$$

The derivations of equations (15) and (16) are shown in Appendix 7.2. The steady-state equilibrium is characterized by the intersection of the above two loci, equations (15) and (16). These two loci are shown in the δ^R-θ plane in Figure 2. Locus (15), along which the workers' outside option value equals the reservation productivity level, slopes upward: a higher θ increases the R.H.S. of equation (15); thus, δ^R has to increase. When θ goes to zero, δ^R approaches b. Intuitively, an increase in market tightness raises a transition rate from unemployment to employment, inducing the outside option value of unemployed workers to rise. As a result, a higher reservation productivity level is required for unemployed workers to accept a job offer.[8]

The locus (16), along which an unfilled vacancy makes zero profit, slopes downward. In contrast to unemployed workers, a rise in market tightness reduces the probability that vacancies will be filled, thereby lowering J^V. Hence, a reservation productivity level falls so as to raise the transition rate of being filled to maintain the zero profit. When θ tends to

[7] When $\alpha = 0$ and $\alpha_v = 0$, $F(\delta^R) + \int_{\delta^R}^{\infty} e^{\alpha(J^E(x)-J^U)}dF(x) = 1$ and $F(\delta^R) + \int_{\delta^R}^{\infty} e^{\alpha_v(J^F(x)-J^V)}dF(x) = 1$. Applying L'Hopitals' Rule to equations (13) and (14) yields the result.

[8] Denote $\Upsilon(\delta^R) \equiv F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}dF(x) \leq 1$. Applying the Leibniz integral rule to differentiate $\ln\Upsilon(\delta^R)$ with respective to δ^R, we have

$$\frac{\partial \ln\Upsilon(\delta^R)}{\partial\delta^R} = -\frac{1}{\Upsilon(\delta^R)}\frac{\alpha\eta(1+r)}{r+\lambda}\int_{\delta^R}^{\infty} e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}dF(x) > 0.$$

Hence, δ^R and θ are positively associated in the locus (15). Following a similar procedure, one could verify that δ^R and θ are negatively associated in the locus (16).

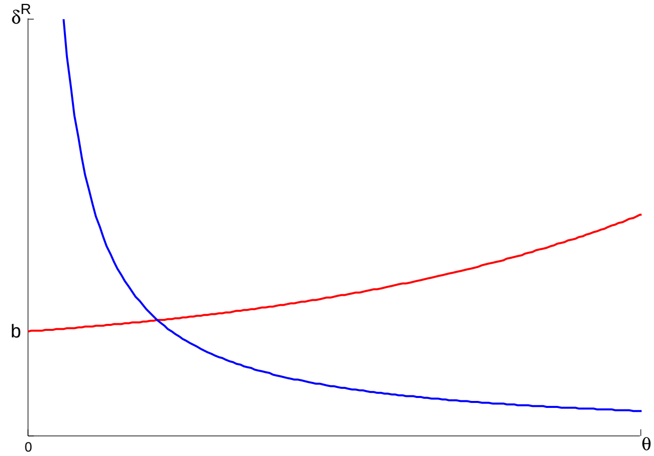


Figure 2: Equilibrium Determination

zero, δ^R approaches infinity. By the continuity of the two functions, the intermediate value theorem ensures that the two loci intersect. Since both loci are strictly monotone, they only intersect once at $\delta^{R*} \in (b, \infty)$ and $\theta^* \in (0, \infty)$. The following proposition summarizes the findings.

Proposition 1. *For all $\alpha \leq 0$ and $\alpha_v \leq 0$, there exists a unique steady-state equilibrium, which is characterized by equations (15) and (16). In the equilibrium, $\delta^{R*} \in (b, \infty)$ and $\theta^* \in (0, \infty)$.*

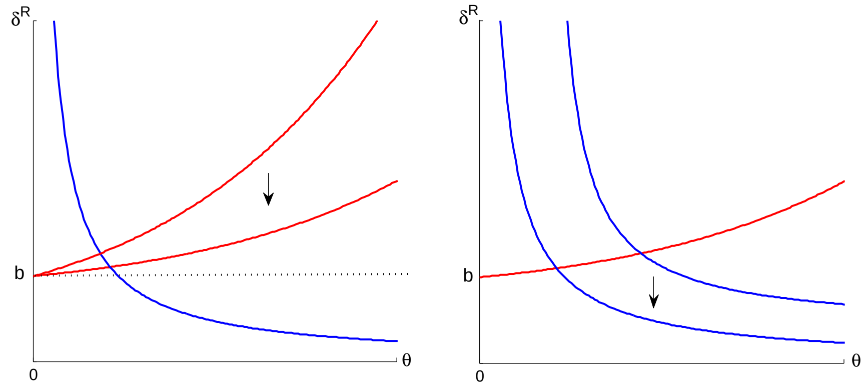
While the literature shows the existence of a unique steady-state equilibrium in the DMP model in the absence of ambiguity aversion, Proposition 1 extends the result to the model with ambiguity preferences: it shows the existence of a unique steady-state equilibrium in the model for all ambiguity preferences $\alpha \leq 0$ and $\alpha_v \leq 0$.

Proposition 2. *In a steady-state equilibrium, (i) θ^* strictly decreases with α and strictly increases with α_v, (ii) δ^{R*} strictly increases with both α and α_v, (iii) $w^*(\delta)$ strictly increases with both α and α_v for all $\delta \geq \delta^{R*}$, and (iv) an unemployment rate strictly increases with α.*

Proof. See Appendix 7.3. □

Figure 3 demonstrates the impacts of the fall in α (the left one) and α_v (the right one) on θ^* and δ^{R*}, respectively. When workers become more ambiguity-averse (i.e., a lower value of α), the locus (15) rotates clockwise and the zero profit condition (16) is unaffected. A new steady-state equilibrium occurs at the intersection of two loci, with a lower value of δ^{R*} and a larger value of θ^* than the old ones.

Figure 3: The Effect of Stronger Ambiguity Aversion on the Reservation Productivity Threshold



Notes: The left (right) panel shows the effect of stronger ambiguity aversion of workers (vacancies).

Intuitively, an ambiguity-averse worker makes a decision based on a distorted model in which a lower likelihood is assigned to a higher match-specific productivity level. With a higher degree of ambiguity aversion, the worker inclines to a more "distorted" model. Therefore, the average ex ante match-specific productivity level is lower, which reduces the outside option value of unemployed workers. Consequently, it requires a lower match-specific productivity level to accept a job offer, causing the reservation productivity level to decline.

From the perspective of vacancy, the reduction in the worker's outside option value means lower wage levels are required to compensate workers with productivity δ for all $\delta \geq \delta^{R*}$, inducing their flow profits and thus raising their expected match surplus. This increase in the expected match surplus incents more supplies of vacancies until the creation of vacancies exhausts the rent in the equilibrium, thereby increasing θ^*. In a steady-state equilibrium, both the fall in δ^{R*} and the rise in θ^* increase the unemployment rate u^* and shorten an unemployment spell.

Similarly, vacancies with stronger ambiguity aversion (a lower value of α_v) think that the average match-specific productivity level and thus expected profits are lower. Hence, the fall in the expected profits reduces the supplies of vacancies, and thus market tightness θ^* declines. The fall in market tightness reduces the transition rate from unemployment to employment; therefore, the worker's outside option value and the reservation productivity level δ^{R*} drop. Consequently, $w^*(\delta)$ declines for all $\delta \geq \delta^{R*}$. While the reduction in the θ^* lengthens an unemployment spell, the fall in δ^{R*} increases the job acceptance rate $1 - F(\delta^{R*})$. Therefore, it is uncertain whether a stronger ambiguity aversion of vacancies increases an unemployment rate. This result can be seen by totally differentiating a steady-state unemployment rate u^* with respect to α_v as follows:

$$\frac{du^*}{d\alpha_v} = \underbrace{\frac{\partial u^*}{\partial \delta^{R*}}}_{+}\underbrace{\frac{d\delta^{R*}}{d\alpha_v}}_{+} + \underbrace{\frac{\partial u^*}{\partial \theta^*}}_{-}\underbrace{\frac{d\theta^*}{d\alpha_v}}_{+}.$$

Importantly, the mechanisms, through which ambiguity preferences affect the labor market, are robust to other wage-setting methods. This paper follows the literature and uses bargained wages that maximize generalized Nash products. Consider an extreme case in which wages are completely independent of the worker's reservation wage and are exogenously given. Given these exogenous wage levels, vacancies with higher degrees of ambiguity aversion believe that lower productivity levels are more likely to be realized. This belief shrinks their expected profit and thus lowers their reservation productivity threshold, reducing the supplies of vacancies. Therefore, ambiguity aversion does affect other labor market variables even though wages are determined not by maximizing the generalized Nash products.

Next, we explore the limiting properties of α and α_v. According to the optimal likelihood ratio (2), for all $\delta > \delta^{R*}$ we have

$$\begin{aligned}\frac{\hat{f}(\delta)}{f(\delta)} &= \frac{e^{\alpha J^E(\delta)}}{\int_0^{\delta^{R*}} e^{\alpha J^U} dF(x) + \int_{\delta^{R*}}^{\infty} e^{\alpha J^E(x)} dF(x)} \\ &= \frac{1}{\int_0^{\delta^{R*}} e^{\alpha(J^U - J^E(\delta))} dF(x) + \int_{\delta^{R*}}^{\infty} e^{\alpha(J^E(x) - J^E(\delta))} dF(x)}.\end{aligned}$$

When α approaches negative infinity, the R.H.S. goes to zero because $J^U - J^E(\delta) < 0$ for all $\delta > \delta^{R*}$. Therefore, the optimal likelihood ratio $\hat{f}(\delta)$ is zero for all $\delta > \delta^{R*}$ and is $f(\delta)/F(\delta^{R*})$ for all $\delta \in [0, \delta^{R*}]$. In other words, if workers are sufficiently ambiguity-averse, they will choose to believe that it is impossible to have any match-specific productivity exceeding the reservation level. If this is the case, the outside option value of workers leaves with unemployment benefit b. Hence, δ^{R*} will approach b: the locus (15) is a horizontal line $\delta^{R*} = b$, as shown in Figure 3. Equivalently, one can show that when α approaches negative infinity, the second term in equation (15) vanishes, and hence, δ^{R*} is equal to unemployment benefit b. The labor market persists: unemployed workers accept all the job offers as long as the match-specific productivity exceeds b.

Interestingly, the labor market collapses when vacancies are sufficiently ambiguity-averse. Applying a similar argument to the optimal likelihood ratio (6), one can show that when α_v approaches negative infinity, the optimal likelihood ratio $\hat{f}_v(\delta)$ is zero for all $\delta > \delta^{R*}$ and is $f_v(\delta)/F(\delta^{R*})$ for all $\delta \in [0, \delta^{R*}]$. In other words, vacancies tend to believe that the likelihood of a match-specific productivity exceeding the reservation level is zero, driving the supply of vacancies and thus market tightness to zero. Equivalently, one can show that when α_v approaches negative infinity, the R.H.S. in equation (16) tends to zero. To maintain the zero profit condition (16), θ^* goes to zero. In this case, the labor market collapses.

Proposition 3. *If $\alpha \to -\infty$, the labor market persists: the reservation productivity level δ^{R*} equals unemployment benefit b. If $\alpha_v \to -\infty$, the labor market collapses: there exists no vacancies.*

Next, we introduce a new measure of unemployment, namely ambiguous unemployment, and investigate its properties. This ambiguous unemployment is defined as the unemployment driven solely by ambiguity preferences. Mathematically, $\tilde{u}(\alpha, \alpha_v) \equiv u(\alpha, \alpha_v) - u(0, 0)$, where $u(\alpha, \alpha_v)$ is the actual unemployment rate under ambiguity preferences α and α_v, and $u(0, 0)$ is the unemployment rate if both workers and vacancies became ambiguity-neutral in reality. We can also define the ambiguous unemployment of workers as $\tilde{u}_W(\alpha, \alpha_v) \equiv u(\alpha, \alpha_v) - u(0, \alpha_v)$, which captures the change in the unemployment rate because of workers' ambiguity preferences. Similarly, the ambiguous unemployment of vacancies is defined as $\tilde{u}_V(\alpha, \alpha_v) \equiv u(\alpha, \alpha_v) - u(\alpha, 0)$.

The ambiguous unemployment of workers can only be negative. According to Proposition 2, an unemployment rate rises if workers are less ambiguity-averse. Hence, $u(\alpha, \alpha_v) < u(0, \alpha_v)$ for all $\alpha < 0$, and thus the ambiguous unemployment of workers $\tilde{u}_W(\alpha, \alpha_v)$ is negative. A negative ambiguous unemployment of workers means the workers' ambiguity preferences reduce the actual unemployment rate. In other words, if workers became ambiguity-neutral in reality, the unemployment rate would have been larger. Whereas conventional wisdom suggests that complete information on the labor market helps job seekers get rid of unemployment, our result reveals that ambiguity aversion towards the productivity distribution indeed reduces the unemployment rate.

Nevertheless, the ambiguous unemployment of vacancies could be any real number. As discussed above, if vacancies become ambiguity-neutral, both θ^* and the δ^{R*} would have been larger. The job finding rate increases, but the job acceptance rate decreases. If the former effect is dominant, $u(\alpha, \alpha_v) < u(\alpha, 0)$. Otherwise, $u(\alpha, \alpha_v) > u(\alpha, 0)$. Therefore, it is uncertain whether the ambiguous unemployment of vacancies $\tilde{u}_V(\alpha, \alpha_v)$ is positive or negative. Because of this indetermination, the ambiguous unemployment $\tilde{u}(\alpha, \alpha_v)$ can also be any real number. We will demonstrate the empirical significance of ambiguous unemployment in Section 5.

Before closing this section, it is important to highlight that job search behaviors are not observationally equivalent under ambiguity aversion and risk aversion. Whereas this section shows that market tightness increases with the degree of the ambiguity aversion of workers, we show in Online Appendix B that a higher degree of risk aversion could lower market tightness. Also, a higher degree of workers' ambiguity aversion reduces unemployment, but the unemployment effect of a higher degree of risk aversion is uncertain.

4 Comparative Statics

The previous section generalizes the DMP model to allow the ambiguity aversion of both workers and vacancies. The DMP model provides generally intuitive comparative statics results that describe the labor market well. This section shows that our generalization preserves most (if not all) of the comparative statics results. In particular, this section investigates how production technology, maintenance costs, unemployment benefits, and matching technology affect other labor market outcomes.

Advances in Production Technology. We first explore the impacts of advances in production technology to the labor market. Consider a permanent productivity shock to the economy so that a productivity distribution F is transformed into a new one G, where the productivity distribution G first-order stochastically dominates F (i.e., $G \succeq_{FOSD} F$).

With the advance in productivity technology, the match-specific productivity level is, on average, higher. Unemployed workers are willing to wait longer for a higher realized productivity level, causing the reservation productivity level to rise. In other words, the outside option value of unemployed workers increases. It is clear from the wage equation (11) that filled vacancies are required to pay more to compensate for the increased outside option value. Therefore, wage levels climb up for all δ exceeding the reservation level.

The impact of the increase in productivity on market tightness is indeterminate. Such an improvement increases the average match-specific productivity level and thus the expected profits. This primary effect induces more supplies of vacancies and thus increases market tightness. Meanwhile, as explained above, such an improvement increases the reservation productivity level, making it harder for unemployed workers to accept a job offer. This general equilibrium effect depresses the supplies of vacancies and thus lowers market tightness. Since we are uncertain about the magnitude of the two forces, the total effect of the advance in productivity distribution on market tightness is indeterminate. As market tightness is one of the key variables in determining a steady-state unemployment rate, the effect on an unemployment rate is also indeterminate. The following proposition summarizes our analytical finding.

Proposition 4. *If F and G are two productivity distributions where $G \succeq_{FOSD} F$, $w_G(\delta) > w_F(\delta)$ for all $\delta \geq \delta_G^R$.*

Proof. See Appendix 7.4. □

Increases in Unemployment Benefit and Vacancy Maintenance Cost. We explore the impact of increased unemployment benefit on the labor market. A rise in unemployment benefit increases the outside option value of unemployed worker. As a result, unemployed workers become more "picky" and require a higher realized productivity level to accept a job offer, thereby increasing the reservation productivity level δ^{R*}. With the increased outside option value, filled vacancies are required to pay higher wage levels to employed workers for all $\delta \geq \delta^{R*}$, depressing profits. With lower expected profits, the zero-profits condition drives some unfilled vacancies out of the labor market in a steady-state equilibrium. Consequently, the supplies of vacancies and thus market tightness θ^* drop. While the rise in δ^{R*} increases the job acceptance rate, the fall in θ^* reduces the likelihood that unemployed workers meet unfilled vacancies. These two effects lead to a higher unemployment rate in a steady-state equilibrium.

Next, we examine the effects of increased vacancy maintenance cost on the labor market. The increased vacancy maintenance cost makes it more costly to create an unfilled vacancy. In other words, a rise in this maintenance cost requires higher expected flow profits to maintain zero profit, discouraging unfilled vacancies to stay in the labor market. Hence, market tightness drops. The decrease in the supplies of unfilled vacancies makes it harder for unemployed workers to meet unfilled vacancies. As a result, the outside option value of workers declines, causing the reservation productivity level and thus wages to fall. Totally differentiating a steady-state unemployment rate with respect to c, we have

$$\frac{du^*}{dc} = \underbrace{\frac{\partial u^*}{\partial \delta^{R*}}}_{+}\underbrace{\frac{d\delta^{R*}}{dc}}_{-} + \underbrace{\frac{\partial u^*}{\partial \theta^*}}_{-}\underbrace{\frac{d\theta^*}{dc}}_{-}.$$

While the fall in θ^* reduces the likelihood that unemployed workers meet vacancies, the lower δ^{R*} makes unemployed workers less picky to job offers. Consequently, the impact of increased maintenance cost on unemployment is indeterminate. Proposition 5 summarizes the findings.

Proposition 5. *A rise in unemployment benefit increases wage levels $w(\delta)$ for all $\delta \geq \delta^{R*}$ and unemployment. A rise in maintenance cost reduces θ^*, δ^{R*}, and $w(\delta)$ for all $\delta \geq \delta^{R*}$.*

Proof. See Appendix 7.5. □

Advances in Matching Technology. An improvement in matching technology increases the matching rate between unemployed workers and unfilled vacancies. A higher matching rate allows both of them to re-search another offer more easily if either side rejects a current offer. Consequently, they become more "picky" during the searching process, driving up the reservation productivity level. Again, a natural consequence is that the wage $w(\delta)$ increases for $\delta \geq \delta^{R*}$ in response to the increase in δ^{R*}. However, the impact on θ^* is uncertain. On the one hand, the rise in a matching rate increases the outside option value of vacancies, thereby providing incentives to create vacancies. On the other hand, the rise in wages reduces expected profits, depressing the supplies of vacancies. Hence, we are uncertain about the effect of the advances in matching technology on θ^* and thus a steady-state unemployment rate. Interestingly, the advances in matching technology do not necessarily imply a lower unemployment rate in a steady-state equilibrium. The following proposition summarizes our finding.

Proposition 6. *If $q_1(\theta) > q_2(\theta)$ for all $\theta \in \mathbb{R}_{++}$, $\delta_1^{R*} > \delta_2^{R*}$ and thus $w_1(\delta) > w_2(\delta)$ for all $\delta \geq \delta_1^{R*}$ in the equilibrium.*

Proof. See Appendix 7.6. □

We have shown analytically how variations in production technology, unemployment benefits, maintenance costs, and matching technology affect the labor market. While our model is shown in Section 3 to nest the canonical one with ambiguity neutrality (i.e., $\alpha = 0$ and $\alpha_v = 0$), this section generalizes the comparative statics results so that all the propositions in this section hold for all $\alpha \leq 0$ and $\alpha_v \leq 0$.

5 Ambiguous Unemployment

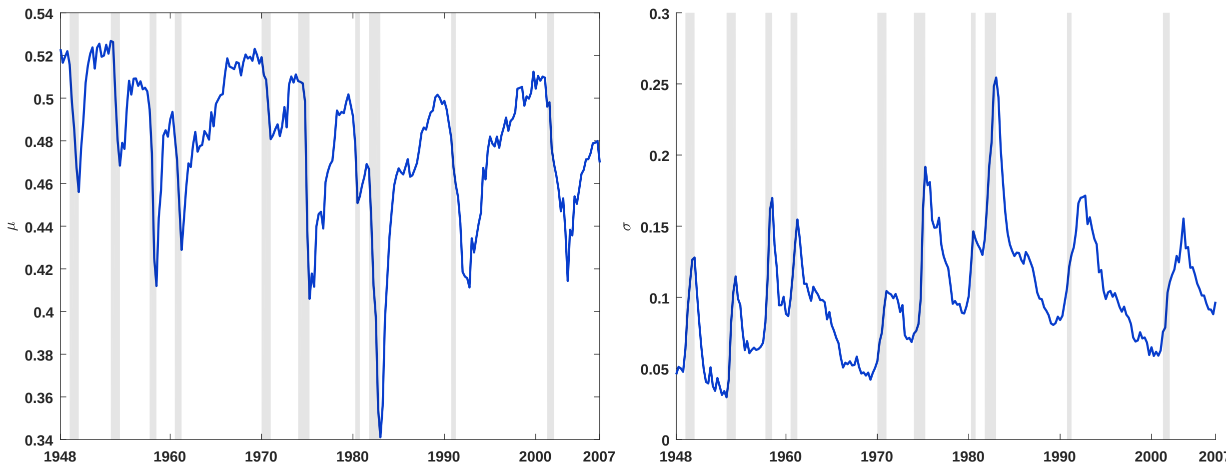
This section quantifies unemployment attributable to ambiguity preferences (i.e., ambiguous unemployment). This analysis is informative for two reasons. First, while Section 3 uncovers the major mechanisms through which ambiguity preferences toward a match-specific productivity distribution affect unemployment, this section measures the empirical significance of these mechanisms. If ambiguity preferences substantially impact unemployment, it will be reasonable to suggest that ambiguity preferences, similar to risk preferences, are important considerations in designing labor market policies. Second, this analysis allows us to compare the sizes of the impacts between the ambiguity preferences of workers and vacancies. And if one is sufficiently larger than the other, this section provides guidance to policymakers to effectively allocate resources in propagating labor market information to workers and firms.

This section uncovers the ambiguous unemployment $\tilde{u}(\alpha, \alpha_v)$ in the United States and decomposes it into the ambiguous unemployments of workers (i.e., $\tilde{u}_W(\alpha, \alpha_v)$) and vacancies (i.e., $\tilde{u}_V(\alpha, \alpha_v)$). We follow the procedure in Appendix 8 to uncover ambiguous unemployment except that we loop over steps 2 and 3 until α and α_v converge to obtain the detection error probabilities. We consider $\alpha = -0.168$ and $\alpha_v = -0.405$, where the detection error probability is about five percent or more.[9]

Next in line is the calibration of all the underlying parameters of steady-state unemployment (12). This unemployment rate is a function of a job separation rate λ and a job finding

[9]The five-percent detection error probability is also used in Croce et al. (2012).

Figure 4: The Calibrated μ and σ



Notes: The left and right panels show the calibrated μ and σ, respectively. The gray areas indicate recession years.

rate $\theta q(\theta)[1 - F(\delta^R)]$. Since we follow Zanetti (2011) to assume that $\delta \sim \ln N(\mu, \sigma^2)$ is log-normally distributed, $F(\cdot)$ is a function of μ and σ. To uncover historical ambiguous unemployment in the United States, we calibrate quarterly u, λ, δ^R, μ, and σ, with other parameters being constant as in Table 2.

These quarterly parameters are calibrated as follows. First, we match u to the actual quarterly unemployment rate provided from Shimer (2012). Second, the separation rates λ are calibrated to the quarterly rate provided from Shimer (2012). Since Shimer (2012) provides these rates from the first quarter of 1948 to the same quarter of 2007, we restrict our analysis to this period. Third, given the actual unemployment rate and the separation rate, the quarterly job finding rate is uniquely pinned down using steady-state unemployment (12). We calibrate δ^R, μ, and σ to ensure that $\theta q(\theta)[1 - F(\delta^R)]$ equals the calibrated quarterly job finding rate. Thus, the two equilibrium conditions (15) and (16) are satisfied.

The calibrated μ and σ cohere with the literature. Figure 4 shows that as average productivity levels tend to be lower in slumps, μ is lower in recession years. This figure illustrates that the calibrated σ is high in slumps, in line with the volatility shocks in recession years documented in Schaal (2017) and Bloom et al. (2018).

Figure 5 demonstrates the counterfactual unemployment rates under ambiguity neutrality. Subtracting these counterfactual unemployment rates from the corresponding actual unemployment rates yields ambiguous unemployment. That is, $\tilde{u}(\alpha, \alpha_v) \equiv u(\alpha, \alpha_v) - u(0, 0)$. This figure indicates that $u(0, 0)$, represented by the dash-dot line, always exceeds the actual unemployment rate, shown by the solid line. This simulation exercise suggests that ambiguous unemployment in the United States is negative in the past half-century, meaning that the ambiguity preferences of workers and vacancies reduce the unemployment rate. In other words, if both workers and vacancies became ambiguity-neutral in reality, the unemployment rate would have been larger.

Next, we explore whether this negative ambiguous unemployment is attributable more to workers' or vacancies' preferences. First, we simulate the counterfactual unemployment rate when workers became ambiguity-neutral (i.e., $u(0, \alpha_v)$). Figure 5 shows that $u(0, \alpha_v)$, represented by the dot line, always exceeds the actual unemployment rate. This result is expected because we show in Section 4 that the ambiguous unemployment of workers is always negative (i.e., $\tilde{u}_W(\alpha, \alpha_v) \equiv u(\alpha, \alpha_v) - u(0, \alpha_v) < 0$.) Second, we simulate $u(\alpha, 0)$ in Figure 5 as well. When comparing between $u(\alpha, \alpha_v)$ and $u(\alpha, 0)$, it is clear that the ambiguous unemployment of vacancies $\tilde{u}_V(\alpha, \alpha_v) \equiv u(\alpha, \alpha_v) - u(\alpha, 0)$ is positive. Probably because of the positive ambiguous unemployment of vacancies, the ambiguous unemployment of workers $\tilde{u}_W(\alpha, \alpha_v)$ is slightly larger than the ambiguous unemployment $\tilde{u}(\alpha, \alpha_v)$ in magnitude.

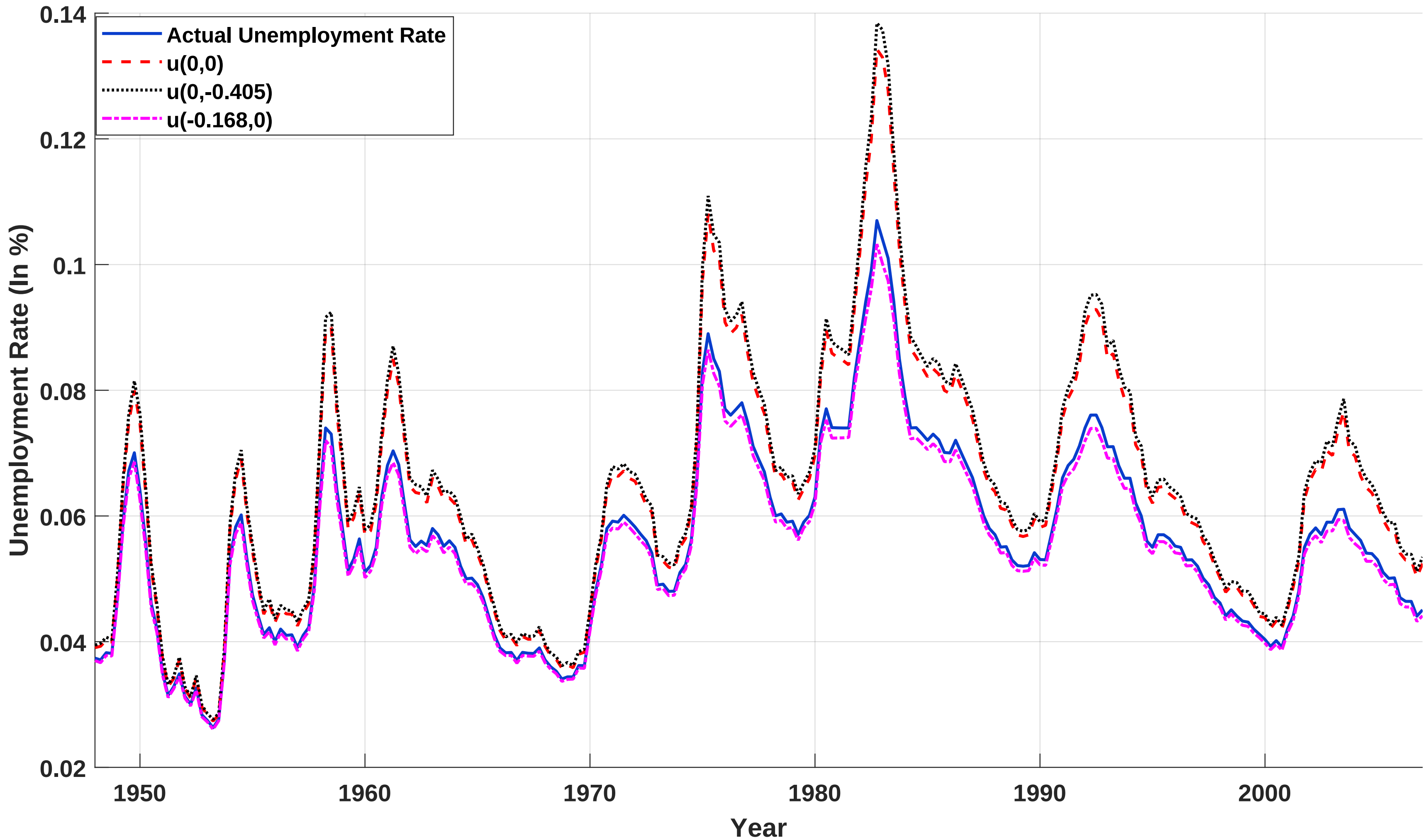
Three points are noteworthy. First, the impact of the ambiguity preference of vacancies on unemployment is larger through the contact rate rather than the job acceptance rate. A stronger ambiguity aversion reduces vacancies' expected profit, reducing the supplies of vacancies and thus increasing unemployment. Meanwhile, the lower expected profit reduces the reservation productivity level δ^{R*}, thereby increasing the acceptance rate and thus decreasing unemployment. Our quantitative exercise suggests that the former effect, which is a primary effect, dominates the latter one, explaining why the unemployment rate is higher under the ambiguity aversion of vacancies.

Second, the impact of the worker's ambiguity preference on unemployment is more pronounced than that of the vacancies. Whereas the ambiguity aversion of workers substantially reduces unemployment via both the job contact rate (through θ) and the job acceptance rate (through δ^R), the ambiguity aversion of vacancies creates two compelling forces on unemployment (as discussed in above). Hence, the effect of workers' ambiguity aversion on unemployment is much larger, explaining the remarkable ambiguous unemployment of workers. This finding highlights the dilemma in allocating resources to propagate labor market information. The removal of the firms' ambiguity reduces unemployment, but the impact is negligible. In contrast, if the workers' ambiguity is removed, the effect is substantial but unemployment increases.

Third, ambiguous unemployment is countercyclical: it is larger in slumps than in booms and reaches its peaks during recession years. This ambiguous unemployment is close to zero in expansion but can reach as high as 19 percent of the actual unemployment in recession years. This finding reveals that ambiguous unemployment can be significant: ambiguous preferences can substantially distort the workers' belief on a productivity distribution and thus unemployment.

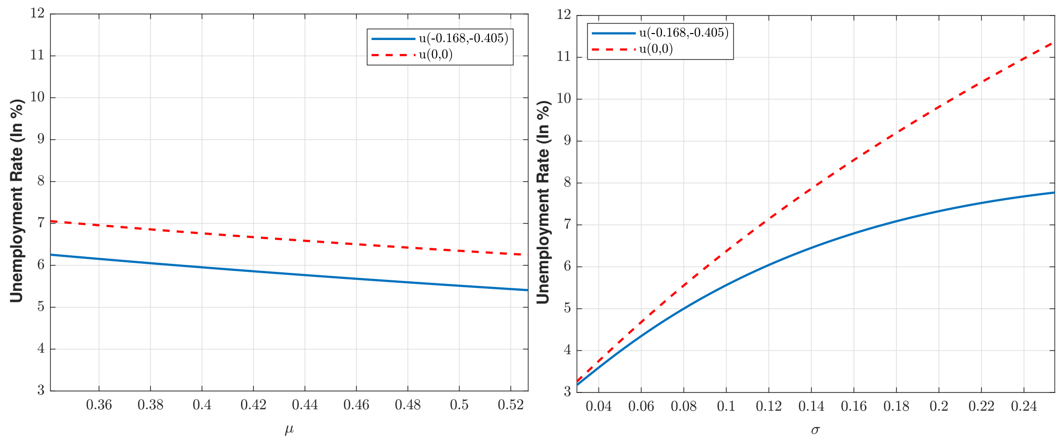
Next, we explore the key driving force for the countercyclical property of ambiguous

Figure 5: Actual and Counterfactual Unemployment Rates in the United States



Notes: The solid line represents the actual quarterly unemployment rate in US. The dash line shows the counterfactual unemployment rate when both workers and vacancies became ambiguity-neutral in reality. The dot (dash-dot) line shows the counterfactual unemployment rate when workers (vacancies) became ambiguity-neutral.

Figure 6: The Effects of μ and σ on Unemployment



Notes: The left (right) panel demonstrates the relationship between μ (σ) and unemployment. The solid and dash lines demonstrate the corresponding relationship with ambiguity aversion and without ambiguity aversion, respectively. Other variables are computed using their averages in the calibration exercise in Figure 5. Along the horizontal axes, μ and σ vary from its minimum to its maximum in Figure 5.

unemployment. Recall from Figure 4 that the mean of the productivity distribution μ is procyclical and its standard deviation σ is countercyclical. In what follows, we investigate whether the countercyclical property of ambiguous unemployment is mainly driven from the procyclicality of μ or the countercyclicality of σ.

To do so, we demonstrate the relationship between an unemployment rate and μ in the left panel of Figure 6. In this exercise, μ varies from its minimum to its maximum in the calibration exercise in Figure 5. θ, δ^R, and u are solved using the steady-state unemployment (12) and the two equilibrium conditions (15) and (16). Other parameters are computed using their averages in Figure 5. In addition to the unemployment rate under ambiguity aversion, we also plot the counterfactual unemployment rate under ambiguity neutrality. Hence, the vertical distance between the two lines captures ambiguous unemployment. Using a similar procedure, the relationship between the unemployment rate and the σ is established in the right panel of Figure 6.

Several points deserve discussion. First, ambiguous unemployment hardly reacts to productivity shock. According to Figure 6, the two unemployment rates fall under productivity shocks; the ambiguous unemployment rate is almost unaffected by productivity shocks. Second, ambiguous unemployment is responsive to the volatility shock to a productivity distribution. According to Figure 6, the unemployment rate under ambiguity aversion increases sharply with the standard deviation of the production distribution, consistent with the literature. While Schaal (2017) finds that the volatility shocks to a productivity distribution can largely explain unemployment fluctuations, Bloom et al. (2018) shows that business cycles are mainly driven by volatility shocks. Strikingly, the counterfactual unemployment rate is much more responsive to this volatility shock. Therefore, ambiguous unemployment increases significantly with the standard deviation of a productivity distribution, and the increase is more pronounced when the standard deviation is higher.

Second, the countercyclicality of ambiguous unemployment is attributable to the volatility shock to a productivity distribution in recession years. While Figure 5, Schaal (2017), and Bloom et al. (2018) indicate that the standard deviation of a productivity distribution increases in recession years, Figure 6 shows that ambiguous unemployment increases sharply with this standard deviation. Hence, the countercyclicality of ambiguous unemployment arises from the volatility shock to a productivity distribution in recession years.

As shown in Figures 4 and 6, Schaal (2017), and Bloom et al. (2018), the standard deviation of a productivity distribution and an unemployment rate are positively correlated. Since the standard deviation of a productivity distribution increases in recession years, and these volatility shocks catalyze ambiguous unemployment, the ambiguity preferences reduce unemployment rates more in slumps than in booms. These findings complement the literature (Bloom, 2009; Schaal, 2017; Bloom et al., 2018) by explaining why the strong correlation between this volatility shock and unemployment is weakened in recession (as shown in Table 1).

Indeed, our findings and Epstein and Schneider (2008) share the same notion in the impact of the ambiguity aversion: ambiguity-averse agents react more strongly to negative shocks than positive shocks because they act as if they take a worst-case scenario. They tend to assign a much higher likelihood to the lower productivity level in an economic downturn because of the increased standard deviation during the same period. In other words, they tend to pick the distorted model that departs more from the approximating model in recession than in expansion. So, the reservation wage would have increased more and market tightness would have decreased more in slumps if workers and vacancies were ambiguity-neutral. Consequently, this stronger volatility shock intensifies ambiguous unemployment in slumps. In other words, the ambiguous preferences reduce the unemployment rate more in slumps because of the stronger volatility shocks, thereby reducing the correlation between this volatility shock and unemployment in recession.

6 Conclusion

This paper constructs a search-theoretical model featuring ambiguity preferences, uncovers the major mechanisms through which ambiguity preferences affect labor market outcomes, quantifies the unemployment attributable to these ambiguity preferences, and resolves the puzzle concerning the relationships between unemployment and volatility shocks to a productivity distribution. It develops an analytically tractable version of the DMP model featuring workers and firms ambiguity preferences. We analytically show that our model preserves most (if not all) of the intuitive comparative statics results in the DMP model, thereby contributing to the broad literature that extends the DMP model to incorporate other labor market features.

Our quantitative analysis contributes to the literature by quantifying ambiguous unemployment and investigating its properties. First, our analysis finds that ambiguous unemployment is negative, suggesting that the ambiguity preferences shrink unemployment. In other words, unemployment would have increased if workers and vacancies became ambiguity-neutral. Second, this ambiguous unemployment rate can be considerable. This unemployment could reach 19 percent of the actual unemployment, calling attention to this ambiguous unemployment. Third, our results point to a policy dilemma in allocating resources in propagating labor market information. If a vacancies' ambiguity toward a productivity distribution is removed, unemployment falls, but the impact is negligible. The removal of a workers' ambiguity has a substantial impact but increases unemployment.

Fourth, ambiguous unemployment is found to be countercyclical. This unemployment is close to zero in booms but reaches 19 percent of the actual unemployment in slumps. Fifth, ambiguity preferences enhance our understanding of the relationship between unemployment and volatility shocks to a productivity distribution. We find that these volatility shocks catalyze ambiguous unemployment: ambiguity preferences reduce unemployment more in slumps because of the stronger volatility shock during the same period, weakening the correlation between unemployment and volatility shocks in recessions.

7 Appendix I: Proof

7.1 Proof of Equation (2)

To solve the minimization problem (1), we can write a Lagrangian function as follows.

$$L = b+\beta\mathbb{E}_x\left[\theta q(\theta)\left(m(x)\max\{J^E(x),J^U\} - \frac{1}{\alpha}m(x)\ln m(x)\right) + (1-\theta q(\theta))J^U\right]+\lambda\left[1-\int m(x)dF(x)\right].$$

The first order condition gives us

$$\theta q(\theta)\left[\max\{J^E(x),J^U\} - \frac{1}{\alpha} - \frac{\ln m(x)}{\alpha}\right] - \lambda = 0.$$

Rearranging terms gives us

$$m(x) = e^{\alpha\max\{J^E(x),J^U\}}e^{-1-\frac{\alpha\lambda}{\theta q(\theta)}}.$$

Integrating both sides over $\mathbb{R}$ and rearranging terms give us

$$\begin{aligned}\int m(x)dF(x) &= \int e^{\alpha\max\{J^E(x),J^U\}}dF(x)e^{-1-\frac{\alpha\lambda}{\theta q(\theta)}} \\ e^{1+\frac{\alpha\lambda}{\theta q(\theta)}} &= \int e^{\alpha\max\{J^E(x),J^U\}}dF(x).\end{aligned}$$

Hence, we have

$$m(\delta) = \frac{e^{\alpha\max\{J^E(\delta),J^U\}}}{\int e^{\alpha\max\{J^E(x),J^U\}}dF(x)}.$$

7.2 Derivations of Equations (15) and (16)

Substituting the wage equation (9) in equation (3) yields

$$J^E(\delta) = (1-\eta)\beta rJ^U + \eta\delta + \beta\Big[\lambda J^U + (1-\lambda)J^E(\delta)\Big].$$

Rearranging terms gives

$$J^E(\delta) - J^U = \frac{\eta(1+r)}{(r+\lambda)}(\delta - \delta^R)$$

Substituting this $J^E(\delta) - J^U$ into equation (13) yields equation (15). Equation (16) can be derived in a similar way.

7.3 Proof of Proposition 2

Assume that $\delta^R \geq b$ is finite. So, $0 < F(\delta^R) \leq 1$. Define

$$B(\alpha) \equiv F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}dF(x)$$

Its first- and second-order partial derivative are given by

$$B'(\alpha) = \frac{\eta(1+r)}{r+\lambda}\int_{\delta^R}^{\infty}(x-\delta^R)e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}dF(x)$$

and

$$B''(\alpha) = \left[\frac{\eta(1+r)}{r+\lambda}\right]^2\int_{\delta^R}^{\infty}(x-\delta^R)^2e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}dF(x).$$

Next, we define

$$\tilde{B}(\alpha) \equiv \frac{1}{\alpha}\ln B(\alpha) > 0 \text{ for all finite } \alpha \leq 0,$$

where $\tilde{B}(\alpha)$ is the last term on the R.H.S. of equation (15). We will show that $\tilde{B}'(\alpha) > 0$ for all $\alpha \leq 0$. The first- and the second-order partial derivatives of $\tilde{B}(\alpha)$ are given by

$$\tilde{B}'(\alpha) = \frac{1}{\alpha}\left(-\frac{1}{\alpha}\ln B(\alpha) + \frac{B'(\alpha)}{B(\alpha)}\right)$$

and

$$\tilde{B}''(\alpha) = -\frac{2}{\alpha}\tilde{B}'(\alpha) + \frac{1}{\alpha}\left(\frac{B''(\alpha)}{B(\alpha)} - \frac{B'(\alpha)^2}{B(\alpha)^2}\right).$$

Lemma 1.

$$\frac{B''(\alpha)}{B(\alpha)} - \frac{B_1(\alpha)^2}{B(\alpha)^2} > 0$$

Proof. Define a function $h(x,\alpha)$ as

$$h(x,\alpha) = \begin{cases} 1 & \text{if } 0 \leq x < \delta^R \\ e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)} & \text{if } x \geq \delta^R \end{cases}$$

Hence, $B(\alpha) = \int_0^\infty h(x,\alpha) f(x)dx$. For any $\alpha \leq 0$, we can define another function

$$g(x,\alpha) \equiv \frac{h(x,\alpha)}{B(\alpha)} f(x).$$

$g(x,\alpha)$ is positive because $h(x,\alpha) > 0$ and $B(\alpha) > 0$ for all $\alpha \leq 0$. Hence, $\int_0^\infty g(x,\alpha) = 1$ and $g(x,\alpha)$ is a probability density function.

$$\frac{B'(\alpha)}{B(\alpha)} = \frac{1}{B(\alpha)}\int_0^\infty \frac{\partial h(x,\alpha)}{\partial\alpha} f(x)dx = \frac{\eta(1+r)}{r+\lambda}\int_0^\infty \max\{x-\delta^R, 0\} g(x,\alpha)dx$$

$$\frac{B''(\alpha)}{B(\alpha)} = \frac{1}{B(\alpha)}\int_0^\infty \frac{\partial^2 h(x,\alpha)}{\partial\alpha^2} f(x)dx = \left[\frac{\eta(1+r)}{r+\lambda}\right]^2 \int_0^\infty \left(\max\{x-\delta^R, 0\}\right)^2 g(x,\alpha)dx$$

Therefore,

$$\frac{B''(\alpha)}{B(\alpha)} - \left(\frac{B'(\alpha)}{B(\alpha)}\right)^2 = \left[\frac{\eta(1+r)}{r+\lambda}\right]^2 Var_x\left(\max\{x-\delta^R, 0\}\right) > 0 \text{ for all finite } \delta^R,$$

where the variance is evaluated with respect to the density function $g(x,\alpha)$. □

Now, we are ready to show that $\tilde{B}'(\alpha) > 0$ for all $\alpha \leq 0$. First, notice that $\lim_{\alpha\to 0}\tilde{B}(\alpha) > 0$ and $\lim_{\alpha\to-\infty}\tilde{B}(\alpha) = 0$.

Second, there exists no $\alpha_0 \leq 0$ such that $\tilde{B}'(\alpha_0) = 0$ and $\tilde{B}''(\alpha_0) \geq 0$. Otherwise, it contradicts to Lemma 1. Hence, if there exists $\alpha_0 \leq 0$ such that $\tilde{B}'(\alpha_0) = 0$, $\tilde{B}''(\alpha_0) < 0$.

Third, there exists no $\alpha_0 \leq 0$ such that $\tilde{B}'(\alpha_0) = 0$ and $\tilde{B}''(\alpha_0) < 0$. Suppose not. Then $\lim_{\alpha\to 0^-} \tilde{'}B(\alpha) < 0$ because $\lim_{\alpha\to 0} \tilde{B}(\alpha) > 0$ and $\lim_{\alpha\to -\infty} \tilde{B}(\alpha) = 0$.

By L'Hopital's rule, $\lim_{\alpha\to 0^-} \tilde{B}(\alpha) = \lim_{\alpha\to 0^-} B'(\alpha)/B(\alpha)$. Hence, we can apply the L'Hopital's rule to evaluate $\lim_{\alpha\to 0^-} \tilde{B}'(\alpha)$. Hence, we have

$$\lim_{\alpha\to 0^-} \tilde{B}'(\alpha) = \lim_{\alpha\to 0^-} \left[-\tilde{B}'(\alpha) + \frac{B''(\alpha)}{B(\alpha)} - \left(\frac{B'(\alpha)}{B(\alpha)}\right)^2 \right],$$

and thus

$$\lim_{\alpha\to 0^-} \tilde{B}'(\alpha) = \lim_{\alpha\to 0^-} \frac{1}{2} \left[\frac{B''(\alpha)}{B(\alpha)} - \left(\frac{B'(\alpha)}{B(\alpha)}\right)^2 \right] > 0.$$

A contradiction arises. Therefore, there exists no $\alpha \leq 0$ such that $\tilde{B}'(\alpha) = 0$. Since $\lim_{\alpha\to 0} \tilde{B}(\alpha) > 0$ and $\lim_{\alpha\to -\infty} \tilde{B}(\alpha) = 0$, $\tilde{B}'(\alpha) > 0$ for all $\alpha \leq 0$.

Therefore, the partial derivative of the R.H.S. of equation (15) with respect to α is positive. A similar argument can be used to show that the partial derivative of the R.H.S. of equation (16) with respect to α_v is positive. Applying Cramer's rule to equations (15) and (16),

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix} \begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} -d\alpha_v \\ -d\alpha \end{pmatrix}$$

Define $A \equiv \det \begin{pmatrix} - & - \\ + & - \end{pmatrix}$.

$$\frac{d\theta}{d\alpha_v} = \frac{\det \begin{pmatrix} - & - \\ 0 & - \end{pmatrix}}{A} > 0, \quad \frac{d\delta^R}{d\alpha_v} = \frac{\det \begin{pmatrix} - & - \\ + & 0 \end{pmatrix}}{A} > 0,$$

$$\frac{d\theta}{d\alpha} = \frac{\det \begin{pmatrix} 0 & - \\ - & - \end{pmatrix}}{A} < 0, \text{ and } \frac{d\delta^R}{d\alpha} = \frac{\det \begin{pmatrix} - & 0 \\ + & - \end{pmatrix}}{A} > 0$$

Using the wage equation (9), simple algebra gives

$$\frac{dw(\delta)}{d\alpha} = (1-\eta)\frac{d\delta^R}{d\alpha} > 0, \text{ and } \frac{dw(\delta)}{d\alpha_v} = (1-\eta)\frac{d\delta^R}{d\alpha_v} > 0.$$

7.4 Proof of Proposition 4

$G \succeq_{FOSD} F$ iff $\int h(x)dG(x) \leq \int h(x)dF(x)$ for any non-increasing function $h(x)$. For any $\delta^R \in \mathcal{R}_+$, we define two non-increasing functions:

$$h(x) = \begin{cases} e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}, & \text{if } \delta > \delta^R; \\ 1, & \text{otherwise.} \end{cases}, \quad \tilde{h}(x) = \begin{cases} e^{\frac{\alpha_v(1-\eta)(1+r)}{r+\lambda}(x-\delta^R)}, & \text{if } \delta > \delta^R; \\ 1, & \text{otherwise.} \end{cases}$$

Hence, equations (15) and (16) can be written as follows:

$$\begin{aligned} \delta^R &= b + \frac{\beta}{\alpha}\ln\left(\int_0^\infty h(x)dF(x)\right) \\ c &= \frac{\beta}{\alpha_v}\ln\left(\int_0^\infty \tilde{h}(x)dF(x)\right) \end{aligned}$$

Define a surjective function $A(\delta^R; a) : \mathcal{R}_+ \times \mathcal{R} \to \mathcal{R}$, with $\partial A(\delta^R; a)/\partial a > 0$. Notice that for any distribution G, there exist a_G and $\tilde{a}_G$ that satisfy the following equations.

$$\begin{aligned} \int_0^\infty h(x)dG(x) + A(\delta_F^R; a_G) &= \int_0^\infty h(x)dF(x) \\ \int_0^\infty \tilde{h}(x)dG(x) + A(\delta_F^R; \tilde{a}_G) &= \int_0^\infty \tilde{h}(x)dF(x) \end{aligned}$$

where δ_F^R is the reservation productivity under the distribution F. Notice that a_G and $\tilde{a}_G$ are unique due the monotonicity of $A(\delta^R; a)$ with respect to a. Without loss of generality, we set $a_F = \tilde{a}_F = 0$ so that $A(\delta^R; 0) = 0$. For any $G \succeq_{FOSD} F$ where $G \neq F$, either $a_G > 0$ or $\tilde{a}_G > 0$ (or both). Consider a distribution $G_0 \succeq_{FOSD} F$ with $a_{G_0} = \tilde{a}_{G_0}$ slightly above zero. Thus, the preceding two equations can be written as follows:

$$\begin{aligned} \delta^R &= b + \frac{\beta}{\alpha}\ln\left(\int_0^\infty h(x)dF(x) - A(\delta^R; a_{G_0})\right) \\ c &= \frac{\beta}{\alpha_v}\ln\left(\int_0^\infty \tilde{h}(x)dF(x) - A(\delta^R; \tilde{a}_{G_0})\right) \end{aligned}$$

To investigate the impact of changing the distribution from F to G_0, it is equivalent to investigate the impact of changing a from 0 to $a_{G_0} = \tilde{a}_{G_0}$. Hence, we apply Crammer's rule to the above two equations,

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix} \begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} -da \\ -da \end{pmatrix}$$

$$\frac{d\delta^R}{da} = \frac{\det\begin{pmatrix} - & - \\ + & - \end{pmatrix}}{A} > 0,\ \frac{d\theta}{da} = \frac{\det\begin{pmatrix} - & - \\ - & - \end{pmatrix}}{A}$$

Thus, $\delta^R_G > \delta^R_F$ if $G \succeq_{FOSD} F$. Hence, $w_G(\delta) = (1-\eta)\delta^R_G + \eta\delta > (1-\eta)\delta^R_F + \eta\delta = w_F(\delta)$ for all $\delta \geq \delta^R_G$. Similarly, applying Crammer's rule to the cases ($a_G > 0$, $\tilde{a}_G = 0$) and ($a_G = 0$, $\tilde{a}_G > 0$) gives the same result.

7.5 Proof of Proposition 5

Applying Cramer's rule to equations (15) and (16),

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix}\begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} +dc \\ -db \end{pmatrix} \tag{17}$$

Denote $A = det\begin{pmatrix} - & - \\ + & - \end{pmatrix} > 0$.

$$\frac{d\theta}{dc} = \frac{\det\begin{pmatrix} + & - \\ 0 & - \end{pmatrix}}{A} < 0,\ \frac{d\delta^R}{dc} = \frac{\det\begin{pmatrix} - & + \\ + & 0 \end{pmatrix}}{A} < 0 \tag{18}$$

$$\frac{d\theta}{db} = \frac{\det\begin{pmatrix} 0 & - \\ - & - \end{pmatrix}}{A} < 0,\ \frac{d\delta^R}{db} = \frac{\det\begin{pmatrix} - & 0 \\ + & - \end{pmatrix}}{A} > 0 \tag{19}$$

7.6 Proof of Proposition 6

If $q_1 = \varphi q$ where $\varphi \in (0,1)$, then $p_1 = \varphi p$. With new matching technology, equations (15) and (16) are written as

$$\delta^R = b + \frac{\beta}{\alpha}\ln\left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha\varphi\theta q(\theta)\eta(1+r)}{r+\lambda}(x-\delta^R)}dF(x)\right) \tag{20}$$

$$c = \frac{\beta}{\alpha_v}\ln\left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha_v\varphi q(\theta)(1-\eta)(1+r)}{r+\lambda}(x-\delta^R)}dF(x)\right) \tag{21}$$

Applying Crammer's rule to the above two equations,

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix}\begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} -d\varphi \\ -d\varphi \end{pmatrix} \tag{22}$$

$$\frac{d\delta^R}{d\varphi} = \frac{\det \begin{pmatrix} - & - \\ + & - \end{pmatrix}}{A} > 0, \quad \frac{d\theta}{d\varphi} = \frac{\det \begin{pmatrix} - & - \\ - & - \end{pmatrix}}{A} \tag{23}$$

Thus, $dw(\delta)/d\varphi > 0$ for all $\delta \geq \delta^R$.

8 Appendix II: Calibration

This section illustrates the procedure to quantify ambiguous unemployment. Since ambiguous unemployment is a counterfactual measure, we quantify it using the following procedure:

1. Calibrate parameters to match an actual unemployment rate under ambiguity neutrality.
2. Compute detection error probabilities using the parameters calibrated in step 1.
3. Choose entropy penalty parameters at the perceived level of the detection error probability.
4. Calibrate parameters to match an actual unemployment rate using the entropy penalty parameters calibrated in step 3.
5. Simulate the counterfactual unemployment rate at $\alpha = 0$ and $\alpha_v = 0$ using the parameters calibrated in step 4.
6. Compute the difference between the counterfactual unemployment rate (from step 5) and the actual unemployment rate.

The difference between the two unemployment rates in step 6 captures ambiguous unemployment (i.e., $\tilde{u}(\alpha, \alpha_v)$). Step 1 is discussed in Subsection 8.1. We calibrate entropy penalty parameters (i.e., α and α_v) following steps 2 and 3 in Subsection 8.2. We will demonstrate how to compute ambiguous unemployment following the rest of the procedure in Subsection 8.3.

8.1 Calibrating the DMP Model Under Ambiguity Neutrality

This subsection follows step 1 to calibrate parameters under ambiguity neutrality (i.e., $\alpha = \alpha_v = 0$). In particular, our calibration exercise matches the model to the U.S. postwar economy (i.e., 1948-2007).

First, we follow Michaillat (2012) to specify a matching function as $M(u, v) = au^{\gamma}v^{1-\gamma}$, in line with the empirical evidence of Petrongolo and Pissarides (2001). Another reason to adopt a Cobb-Douglas matching function is to ensure that the elasticity of $q(\theta)$ with respect to θ is constant. In this way, we can calibrate this elasticity γ to a worker's bargaining power η so that the Hosios condition holds. Following Shimer (2005), the worker's bargaining power η is set to be 0.72. We follow Hagedorn and Manovskii (2008) to set market tightness θ to be 0.634.

We match the model period to be a quarter. We set $\lambda = 0.0344$, which is the average of the separation rate data provided by Shimer (2012). In Shimer (2005), the quarterly job finding rate is 1.35, implying that the matching technology $a = 1.35/0.634^{1-0.72} = 1.534$. The quarterly unemployment rate data from 1948 to the first quarter of 2007 are obtained from the Bureau of Labor Statistics, and its average is about 0.056. We follow Shimer (2005) to set a quarterly interest rate to be 0.012.

Next in line is the productivity distribution. This exercise follows Zanetti (2011) to assume that a match-specific productivity distribution is log-normally distributed. This calibration strategy coheres with the empirical evidence of Lydall (1968) and Heckman and Sedlacek (1985) in that a wage distribution has a unique interior mode with a log-normal-like skewness. Hence, $\delta \sim \ln N(\mu, \sigma^2)$, where the mean μ and the standard derivation σ are two parameters we calibrate.

Calibrating unemployment benefit and vacancy posting cost are challenging. Shimer (2005) sets the unemployment benefit to be 0.4. However, Hagedorn and Manovskii (2008) argue that this value is too low as it does not include forgone leisure or home production. They calibrate their model to match the cyclical properties of wages; nevertheless, their result $b = 0.955$ is implausibly large. Mortensen and Nagypal (2007) argue that if $b = 0.955$, the flow surplus of the employed will be too low. Hence, we follow Hall and Milgrom (2008) and Pissarides (2009) to set $b = 0.71$. This value is larger than 0.4 because it includes the consumption difference between states of employment and unemployment. In Hagedorn and Manovskii (2008), the costs of posting a vacancy include a non-capital hiring cost of 0.110 and an idle capital cost 0.474, amounting to 0.584. Therefore, c is set to be 0.584.

We calibrate the mean μ and the standard deviation σ of a productivity distribution together with the equilibrium θ^*, δ^{R*}, and u^*. Given μ and σ, the three variables are pinned down by the three equilibrium conditions (12), (15), and (16). Hence, we solve $\theta^* = 0.634$, $u^* = 0.056$, and the three equilibrium conditions to obtain (μ, σ, u, θ, δ^R). Table 2 summarizes the results.

8.2 Calibrating Entropy Penalty Parameters

This subsection executes steps 2 and 3 to calibrate the two entropy penalty parameters α and α_v. We closely follow the calibration strategy suggested by Hansen and Sargent (2008). First, we map α and α_v to detection error probabilities for discriminating between an approximating model and a chosen worst model associated with the corresponding parameter

Table 2: Endogenous Variables for Calibration

Panel A: Endogenous Variables for Calibration			
Variable	Value	Description	
θ	0.634	Market Tightness	
δ^R	1.662	Reservation Productivity Level	
u	0.056	Unemployment Rate	
Panel B: Calibrated Parameters			
Parameter	Value	Description	Target
μ	0.4935	Mean of a Productivity Distribution	$\theta = 0.634$
σ	0.0822	Standard Deviation	$u = 0.056$
Parameter	Value	Description	Source
r	0.012	Quarterly Interest Rate	Shimer (2005)
λ	0.0344	Separation Rate	Shimer (2012)
b	0.71	Unemployment Benefit	Hall and Milgrom (2008)
η	0.72	Worker's Bargaining Power	Shimer (2005)
a	1.534	Technology of a Matching Function	Shimer (2005)
c	0.584	Cost of Posting a Vacancy	Hagedorn and Manovskii (2008)

values. To compute the detection error probabilities, other parameter values are obtained from Table 2. Second, the detection error probability is used to determine α and α_v.

We first brief the procedure for estimating the detection error probability.[10] A likelihood ratio test is used to compute this detection error probability. Consider two alternative models: model A and model B are an approximating model and an distorted model, respectively. Denote L_j as the likelihood function of a corresponding model $j \in \{A, B\}$. The test suggests that a worker will pick model A iff $L_A > L_B$; otherwise, model B is selected. Given that model j generates the data, a detection error probability is $\Pr(L_j < L_{-j}|j)$. Intuitively, it is a probability of choosing a wrong model $-j$ when the underlying model is j. Set the prior probability of model A and B as one-half. A detection error probability is given by

$$\Pr(\alpha) = \frac{1}{2}\Bigg(\Pr(L_A < L_B|A) + \Pr(L_A > L_B|B)\Bigg).$$

In this calibration exercise, we generate $10,000$ samples for each α. In each sample, 200 observations of wage are generated from the approximated model $\delta \sim \ln N(\mu, \sigma)$. $\mu = 0.4935$ and $\sigma = 0.0822$ are acquired from Table 2, and the distorted model is described in equation (2). Since the model period is about a week, the 200 observations of wage are equivalent to the data of four years. Likelihood functions L_A^i and L_B^i are computed for each sample. With $N(\mu, \sigma^2)$ and $w(\delta) = (1-\eta)\delta^R + \eta\delta$, the likelihood function of the

[10]Readers who are interested in the details are referred to Chapters 9 and 10 in Hansen and Sargent (2008).

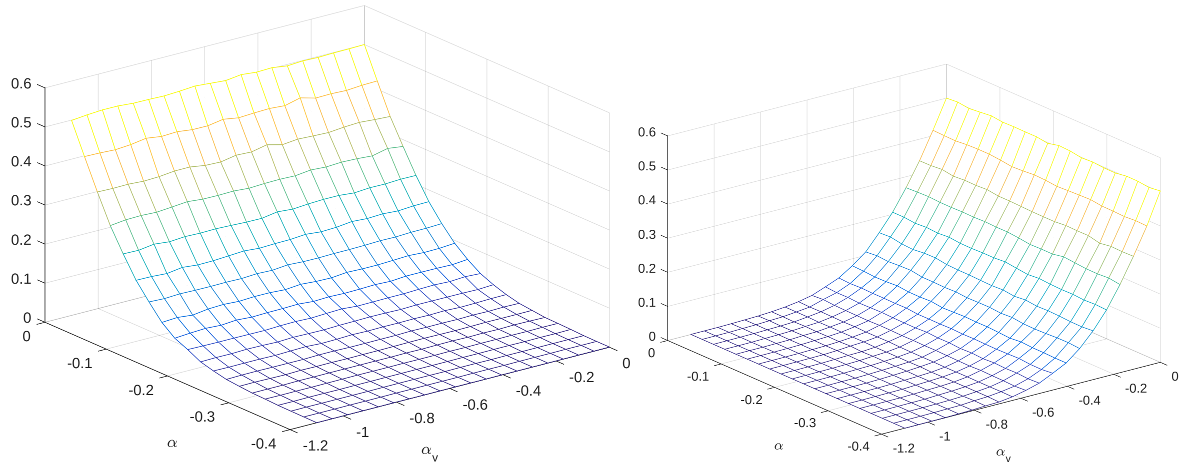


Figure 7: The left (right) panel shows the detection error probability as a function of ambiguity aversion of workers (vacancies).

approximated model is given by

$$L_A^i = \frac{1}{1-F(\delta^R)} \left(\prod_{t=1}^{200} \frac{\eta}{(w_t^i - (1-\eta)\delta^R)\sqrt{2\pi}\sigma} e^{-\frac{\left(\ln(w_t^i-(1-\eta)\delta^R)-\ln\eta-\mu\right)^2}{2\sigma^2}} \right),$$

where $F(\cdot)$ is a cumulative distribution function of $\ln N(\mu, \sigma^2)$, and w_t^i is the t-th observation of wage in a sample i. L_B^i can be calculated in a similar way based on the conditional density given by equation (2), in which $\hat{f}(\delta|\delta > \delta^R) = \hat{f}(\delta)/(1-\hat{F}(\delta^R))$. Noting that $\hat{F}(\cdot)$ is a cumulative distribution function of $\hat{f}(\cdot)$, and $\hat{F}(\delta^R) = e^{\alpha J^U} F(\delta^R)/\int_0^\infty e^{\alpha \max\{J^E(x), J^U\}} dx$. $\Pr(L_j < L_{-j}|j)$ equals $\sum_{i=1}^N I(L_j^i < L_{-j}^i)/N$, where $N = 10,000$ is the number of samples and $I(\cdot)$ is an indicator function to count the number of samples in which the worker picks a wrong model.

In our model, workers and vacancies are ambiguity-averse; hence, we compute the detection error probabilities $p(\alpha)$ and $p(\alpha_v)$. When the entropy penalty parameter is zero, the corresponding approximating model and distorted model are identical. Since the two models cannot be distinguished, the detection error probability will be 0.5. As the entropy penalty parameter falls, the two models are more distinguishable and the detection error probability declines.

Two points deserve mentioning. First, the detection error probability $p(\alpha)$ may depend on α_v, and analogously $p(\alpha_v)$ may depend on α. Figure 7 indicates such a dependence exists but is weak. Second, there could be multiple pairs of α and α_v to yield the same detection error probability. For each detection error probability, we take the average of the α that yields the same detection error probability for all α_v.

Although α_v is unbounded, $p(\alpha_v)$ decays sharply to zero. Hence, we confine the range of α_v by setting its lower bound to -1.1 because the detection error probabilities of vacancies are close enough to zero for all α as $\alpha_v = -1.1$. Here, we implicitly presume that the detection error probability schedules in Figure 7 are common knowledge to workers and vacancies so that workers know that α_v is likely to fall in the interval $[-1.1, 0]$. We follow this procedure to obtain all the detection error probabilities corresponding to α, and we can acquire all the detection error probabilities corresponding to α_v using a similar procedure.

Here, α can be interpreted as a worker's concern about the robustness of the approximating model with the detection error probability $p(\alpha)$. According to Figure 7, workers with $\alpha = -0.18$ are those who pick the distorted model such that the probability of a misspecification error is about five percent. If a reasonable preference for robustness is the rule that functions well for alternative models with the detection error probabilities of five percent or more, $\alpha = -0.18$ will be the choice of the parameter. Similarly, vacancies with $\alpha_v = -0.46$ will pick the distorted model to have the likelihood of a misspecification error equal to five percent.

8.3 Measuring Ambiguous Unemployment

This subsection follows the rest of the procedure to uncover ambiguous unemployment. Notice that plugging the calibrated parameters in Table 2 into the DMP model with ambiguity neutrality will yield the unemployment rate equal to the actual unemployment rate. Hence, we follow step 4 to calibrate the DMP model with ambiguity preferences (i.e., $\alpha = -0.18$ and $\alpha_v = -0.46$) to the U.S. economy using the a similar procedure as discussed in Subsection 8.1. That is, $u(-0.18, -0.46) = 0.056$.

Using the new set of calibrated parameters, we can simulate the counterfactual unemployment rate in the model with ambiguity neutrality, giving us $u(0, 0)$. This is step 5. Finally, we execute step 6 to compute the difference between $u(\alpha, \alpha_v)$ and $u(0, 0)$, which is the ambiguous unemployment. Of course, we can acquire the ambiguous unemployment of workers by simulating $u(0, \alpha_v)$ in step 5 and computing $u(\alpha, \alpha_v) - u(0, \alpha_v)$ in step 6. A similar procedure yields ambiguous unemployment of vacancies. Section 5 follows these six steps to uncover ambiguous unemployment in the United States.

References

Acemoglu, D. (2001). Good Jobs versus Bad Jobs. *Journal of Labor Economics 19*(1), 1–21.

Ahn, D. S. (2007). Hierarchies of Ambiguous Beliefs. *Journal of Economic Theory 136*(1), 286–301.

Albrecht, J., M. Robayo-Abril, and S. Vroman (2018). Public-Sector Employment in an Equilibrium Search and Matching Model. *Economic Journal*.

Bloom, N. (2009). The Impact of Uncertainty Shocks. *Econometrica 77*(3), 623–685.

Bloom, N., S. Bond, and J. Van Reenen (2007). Uncertainty and Investment Dynamics. *Review of Economic Studies 74*(2), 391–415.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. Terry (2018). Really Uncertain Business Cycles. *Econometrica*.

Cairo, I. and T. Cajner (2018). Human Capital and Unemployment Dynamics: Why More Educated Workers Enjoy Greater Employment Stability. *Economic Journal*.

Calvó-Armengol, A. and Y. Zenou (2005). Job matching, Social Network and Word-of-Mouth Communication. *Journal of Urban Economics 57*(3), 500–522.

Croce, M. M., T. T. Nguyen, and L. Schmid (2012). The Market Price of Fiscal Uncertainty. *Journal of Monetary Economics 59*(5), 401–416.

Dolado, J. J., M. Jansen, and J. F. Jimeno (2009). On-the-Job Search in a Matching Model with Heterogeneous Jobs and Workers. *Economic Journal 119*(534), 200–228.

Epstein, L. G. and M. Schneider (2008). Ambiguity, Information Quality, and Asset Pricing. *Journal of Finance 63*(1), 197–228.

Fujita, S. and G. Ramey (2012). Exogenous versus Endogenous Separation. *American Economic Journal: Macroeconomics 4*(4), 68–93.

Galenianos, M. (2014). Hiring Through Referrals. *Journal of Economic Theory 152*, 304–323.

Gonzalez, F. M. and S. Shi (2010). An Equilibrium Theory of Learning, Search, and Wages. *Econometrica 78*(2), 509–537.

Hagedorn, M. and I. Manovskii (2008). The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited. *American Economic Review 98*(4), 1692–1706.

Hall, R. E. (2005). Employment Fluctuations with Equilibrium Wage Stickiness. *American Economic Review 95*(1), 50–65.

Hall, R. E. and P. R. Milgrom (2008). The Limited Influence of Unemployment on the Wage Bargain. *American Economic Review 98*(4), 1653–1674.

Hansen, L. P. and T. J. Sargent (2008). *Robustness*. Princeton University Press.

Heckman, J. J. and G. Sedlacek (1985). Heterogeneity, Aggregation, and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market. *Journal of Political Economy 93*(6), 1077–1125.

Lydall, H. (1968). *The Structure of Earnings*. Clarendon Press.

Michaillat, P. (2012). Do Matching Frictions Explain Unemployment? Not in Bad Times. *American Economic Review 102*(4), 1721–1750.

Mortensen, D. T. and E. Nagypal (2007). More on Unemployment and Vacancy Fluctuations. *Review of Economic Dynamics 10*(3), 327–347.

Mortensen, D. T. and C. A. Pissarides (1994). Job Creation and Job Destruction in the Theory of Unemployment. *Review of Economic Studies 61*(3), 397–415.

Moscarini, G. (2005). Job Matching and the Wage Distribution. *Econometrica 73*(2), 481–516.

Nishimura, K. G. and H. Ozaki (2004). Search and Knightian Uncertainty. *Journal of Economic Theory 119*(2), 299–333.

Papageorgiou, T. (2014). Learning Your Comparative Advantages. *The Review of Economic Studies 81*(3), 1263–1295.

Petrongolo, B. and C. A. Pissarides (2001). Looking into the Black Box: A Survey of the Matching Function. *Journal of Economic Literature 39*(2), 390–431.

Pissarides, C. A. (2000). *Equilibrium Unemployment Theory*. The MIT Press.

Pissarides, C. A. (2009). The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer? *Econometrica 77*(5), 1339–1369.

Postel-Vinay, F. and H. Turon (2014). The Impact of Firing Restrictions on Labour Market Equilibrium in the Presence of On-the-job Search. *Economic Journal 124*(575), 31–61.

Rogerson, R., R. Shimer, and R. Wright (2005). Search-Theoretic Models of the Labor Market: A Survey. *Journal of Economic Literature 43*(4), 959–988.

Rosén, Å. (2003). Search, Bargaining, and Employer Discrimination. *Journal of Labor Economics 21*(4), 807–829.

Sasaki, M. (1999). An Equilibrium Search Model with Coworker Discrimination. *Journal of Labor Economics 17*(2), 377–407.

Schaal, E. (2017). Uncertainty and Unemployment. *Econometrica 85*(6), 1675–1721.

Shimer, R. (2005). The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *American Economic Review 95*(1), 25–49.

Shimer, R. (2010). *Labor Markets and Business Cycles*. Princeton University Press.

Shimer, R. (2012). Reassessing the Ins and Outs of Unemployment. *Review of Economic Dynamics 15*(2), 127–148.

Vindigni, A., S. Scotti, and C. Tealdi (2014). Uncertainty and the Politics of Employment Protection. *Journal of Labor Economics 33*(1), 209–267.

Zanetti, F. (2011). Labor Market Institutions and Aggregate Fluctuations in a Search and Matching Model. *European Economic Review 55*(5), 644–658.